# Nemeth Code Made Easy <br> Christopher Weaver 

## Preface

This text is intended to give the reader a basic feel for the structure of the Nemeth Braille Code for Mathematics and Scientific Notation 1972 Revision (Nemeth, 1972). We will tour the basic structure of the Nemeth Code from a design standpoint. While on this tour we will observe and take advantage of the Nemeth Code's cognitively natural correspondence with print mathematics notation. In the process we will learn MathSpeak. MathSpeak is Dr. Nemeth's method of reading mathematics aloud so that it can be written down while the reader is speaking in either Nemeth Code or print. In fact, it conforms exactly to the structure of the Nemeth Code! In previous attempts to introduce these concepts I separated MathSpeak as a prerequisite to the braille structures under consideration. Practice has shown that they function better in tandem. Thus, all structures will be explained in terms of MathSpeak and braille cell notation simultaneously, and examples shall be in the form:

## Example

- Print Math: $\frac{x+y^{2}}{k+1}$
- MathSpeak: "bfrac x plus y sup 2 base over k plus one efrac"
- Braille: :0: : : : : : : : : : : : : ! : : : : : : : :

Don't worry, we will see this example again, but in its proper place in the discussion where it won't look so daunting. In fact, by the time we are done, not much of secondary math notation will look daunting anymore.

## Disclaimer

This pamphlet is neither a complete nor a correct version of the Nemeth Code. It is incomplete because the Nemeth Code specification is too large for this document to encompass all the gory details. It is not correct because it side steps the issue of integrating math with grade II braille so as to remain accessible to someone who has never seen braille before. I have, thus, left out any reference to single letters and single letter indicators. I encourage a more detailed inspection of the specification itself once you understand the basic design of the code. Also, for this document we will limit our focus to structural math as would be encountered in pre-algebra and above.

## Design

Print mathematics notation has developed over several centuries with the subtextual objective of allowing the reader to precisely analyze all given expressions visually. Since mathematical structures occur embedded within one another in something of a tree like fashion, print notation has evolved to make this structure obvious to the reader. To this end mathematics notation employs several shifts in typeface and symbol size and position within a given expression. We refer to these shifts as "implicit notational events", the
explicit notational events being the symbols themselves or other significant glyphs such as fraction lines. For example, see how the following print notation makes the corresponding tree structure obvious.

$$
\frac{1}{\sqrt{\pi}} e^{-x^{2}}
$$



Note how the implicit notational events guide your eye to the correct tree structure. The process that you go through as you analyze the tree structure is called parsing, and being able toy do it correctly is absolutely essential to your understanding of the mathematical content represented by that print expression. As one can imagine using print like shifts in braille won't work. Fingers do not have the eye's capacity to scan two dimensionally. Thus dots that get shrunk and raised just look like a damaged braille page. In his contribution to Braille for the Millennium Dr. Nemeth (2001) outlined his design principles for the Nemeth Code. As some of them are more faithfully implemented in the Unified Braille System, an alternative to the proposed Unified English Braille Code, I have selected the few for presentation that I think best characterize the 1972 revision. They are quoted editorially as follows:

1) Non Enclosure Principle: Don't put any phantom parentheses or other enclosures into the braille that are not in the print.
2) Just In Time Information Principle: Give the reader the needed information exactly when it is needed--Don't make him/her have to go looking for it
3) Be True To The print Principle: Don't make any braille notation that does not correspond to the print notation. I.E. don't make the braille notation dependant on the meaning of an expression; let the reader infer that from his/her knowledge base just as a print reader would.
4) Good Mnemonics Principle: Give the reader symbols that are grouped together logically when the print symbols are grouped together logically. Preserve Symmetry of notation.
5) Continuous Notation Principle: Don't interrupt the reader's reading with letter signs and number signs unless some specific indication is needed.

We will explore these ideas in detail, as they become relevant to us.

## Braille Basics

At the level of cells Nemeth code looks like any other six-cell braille. As I am hoping that this pamphlet will be shared with traditional teachers of mathematics, I felt it necessary to
include a pictorial introduction to the basics of braille. However, this document assumes that the reader has an understanding of the basic operation of a Perkins Braillewriter, and it shall only provide a gloss of the alphabet and basic symbols of operation. All other braille cells will be provided just in time, as they are needed for the discussion.

Basic Cells

| Letter/digit |  | b |  | d | e | f | g | h |  |  |  |  | m | N | O | p | q |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cell | - | : | $\because$ | $\because:$ | $\because$ | $\because:$ | :: | : | $\because$ | : | $\because$ | : | $\because$ | $\because:$ | $\because$ | !: | :: | !: | : |  | $\because$ |
| Letter/digit | v | W | X | y | z | 0 | 1 | 2 | 3 | 4 |  | 6 | 7 | 8 | 9 | \# |  | + |  |  |  |
| Cell | $\vdots$ | :: | $\because:$ | -: | $\cdots$ | : | - | : | : | $\because:$ | : | :! | :: | : | $\because:$ | : | - | - | $\because$ | : |  |

## Basic In-Line Math Notation

You may recall that numbers are written with a number sign if they are preceded immediately by a space. As you may have noticed, the dot patterns for the numbers are exactly the same as the letters a-j except top two rows have been shifted to the bottom two rows. Thus the number sign serves as a level indicator.

## Example

- Print Math: 12
- MathSpeak: "one two"
- Braille: :: : : : :

The number sign is ONLY used when an indication of level is needed as per the Continuous Notation Principle. Thus, we do not articulate it unless we need to correct a student's forgetfulness. Now when we read numbers or symbols of operation aloud, we articulate each of the digits. Similarly when we read letters in an expression aloud we articulate each letter separately. If we have capitalized letters we say "cap" before each capitalized letter. If we have Greek letters, then we say "Greek" before the letter. The Greek indicator in braille is $!$. If we have a Greek Capitalized letter then we say "Greek cap" before the letter in that order.

## Example

- Print Math: $12+\mathrm{x}$
- MathSpeak: "one two plus x"
- Braille: : : : : : : : : : :


## Example

- Print Math: $12+\Xi$
- MathSpeak: "one two plus Greek cap Gzigh"
- Braille: :! :! : : : : : : : : : : :

Normally Greek letters have obvious correspondences with roman letters, i. e. $\alpha$ is assigned to a. But there are exceptions. Here is a brief table of the exceptions

| Letter | Capital Letter | MathSpeak | Cell |
| :--- | :--- | :--- | :--- |
| $\eta$ | H | Eta | $\vdots \vdots$ |
| $\theta$ | $\Theta$ | Theta | $\vdots \vdots$ |
| $\xi$ | $\Xi$ | Gzigh | $\vdots \vdots$ |
| $v$ | Y | Upsilon | $\vdots \vdots$ |
| $\chi$ | X | Chi | $\vdots \vdots$ |
| $\psi$ | $\Psi$ | Psi | $\vdots \vdots$ |
| $\omega$ | $\Omega$ | Omega | $\vdots!$ |

Since there is a little bit of visible space in the following example, we indicate that with a vocal pause that is reflected in the braille.

## Example

- Print Math: $\cos x$
- MathSpeak: "c o s (pause) x"
- Braille: :0: ! : ! !

Similarly, if there is a coma or a dot we articulate that. If we encounter a negative number that requires a number sign, then the negative sign comes before the number sign.

In the case of multiplication there are a number of signs used in print. Because of this, we must be true to the print as per principle number three. In the case where the multiplication is implicit we simply say the numbers or letters being multiplied without any explicit reference to the multiplication.

## Example

- Print Math: $5 x$
- MathSpeak: "five x"

Sometimes we see a $\times$ indicating multiplication as is common in pre-algebra courses. We call this "cross times"


## Example

- Print Math: $5 \times 3$
- MathSpeak: "five cross-times three"


However, as students progress through Algebra we more commonly see a ${ }^{\bullet}$ as the multiplication indicator. We call this "dot-times"

## Example

- Print Math: $5 \times 3$
- MathSpeak: "five dot-times three"

In Pre-Algebra $\mathrm{a} \div$ is used to indicate simple division tasks. We call this "inline-divide".


## Example

- Print Math: $5 \div 3$
- MathSpeak: "five inline-divide three"

Although I discourage my print students from using slash notation for division because it is inherently ambiguous, it is simply read as "slash" if encountered in a text.


## Example

- Print Math: 5/3
- MathSpeak: "five slash three"

If you encounter any signs that I have not treated in this text I refer you to the code specification (Nemeth, 1972). http://www.tsbvi.edu/math/nemeth-reference.htm is an excellent cheat sheet prepared by Susan Osterhaus at the Texas School for the Blind and Visually Impaired.


## Exercises

Say and braille the following:

1. $1+\mathrm{x}+\mathrm{y}$
2. 2.5-6.7
3. $-298,475$
4. $\sin \theta+\sin \alpha$
5. $28.6 \times 5 \div x$

## Signs of Comparison

One of the basic structures in print mathematics is the equals sign. In fact, you can't technically have an equation without one. When we see an equals sign similar notation such as a greater than sign, or a less than or equal to sign, we simply repeat that sign as if it were in English. If you want to you can abbreviate less than or equal to by saying
"less-eek" be sure to explain what you are doing to your student, however. In braille these signs are four or five cell constructions. The first cell in the construction is always a space, and the last cell is always a space. This is to distinguish them from Greek letters and the like. For instance, the equals sign without the surrounding spaces would be the Greek letter kappa! Therefore, numbers that come after a sign of comparison need the level indicating number sign. Here, I was planing to put a table of the signs, but to see how they work with the spaces, it is best to just go through some examples.

## Example

- Print Math: $x=1$
- MathSpeak: "x equals one"
- Braille: ! : ! : : $\vdots:$

Note the number sign on the braille 1.

## Example

- Print Math: $x<y$
- MathSpeak: "x is less than $y$ "
- Braille: :! : ! :

Note the similarity between the print less-than sign and the braille one. Greater than is similar. Now if we want to express the or equal to part, we stick the cell $\vdots$ : before the last space.

Example

- Print Math: $2 x+5 \geq 6$
- MathSpeak: "two x plus five is greater than or equal to 6 "

Be careful with the greater than sign because out of context it looks like a ". 1 ".
I will leave you the exercises to discover what less than or equal to, and greater than by itself look like. I defer a discussion of more complicated signs of comparison to the Nemeth Code book as they appear very rarely in secondary math.


## Exercises

Say and braille the following:

1. $2 x=3$
2. $y-7>6$
3. $-1 \leq \cos x \leq 1$
4. $5>x \geq 2$
5. $\kappa=\kappa$ (I couldn't resist!)

## Enclosures

The last things that we need to be able to do any in-line math are signs of enclosure. There are three that are commonly used: () parentheses, [] brackets, and \{\} braces. As there is a left and a right form of each of these, when we are speaking them we say "lparen" for left parenthesis and "r-paren" for a right parenthesis. Similarly we say things
like "l-bracket" or "r-brace". Don't refer to them as "open parens" because mathematicians have this bad habit of opening an enclosure with a right parenthesis and closing it with a left bracket as a means of pure notational abuse. The l-paren is the cell :! and the r-paren is the cell $: \vdots$. We make brackets and braces by sticking a bracket or brace indicator before a respective 1-paren or r-paren. As pat of the Just in Time
Information Principle these indicators always come before, even though it would be more visually balanced for them to come after. So the cell patterns are:

| $[$ | $]$ | $\{$ | $\}$ |
| :--- | :--- | :--- | :--- |
| $\vdots \vdots:$ | $\vdots \vdots:$ | $\vdots \vdots:$ | $\vdots \vdots \vdots$ |

Let's see some examples of these enclosures in action!
Example

- Print Math: $(2 x+5)$
- MathSpeak: "l-paren two x plus five r-paren"
- Braille: :: :: :0 : : :! : : :


## Example

- Print Math: 3[6-(2x+5)]
- MathSpeak: "three l-bracket six minus l-paren two x plus 5 r-paren r-bracket"



## Example

- Print Math: $\{1, a, c o w\}$
- MathSpeak: "l-brace one comma a comma c o w r-brace"

Note the spaces added so as not to confuse the reader regarding commas in a number versus commas separating list items. Number signs need to be used in the case that numbers with spaces before them are in the group.


## Example

- Print Math: $] 0,1$ ]
- MathSpeak: "l-bracket zero comma 1 1-bracket"
- Braille: : : : : : : : : : : : : : :

Again, if you encounter more complicated signs of enclosure, I refer you to the Nemeth Code Book or to the TSBVI site.

## Exercises

Say and braille the following:

1. $(2 x-1)$
2. $1-[1-(1-x)]$
3. $\{a, a a, a a a, b, a b, a a b, a a a b\}$

## Structural Math Notation

## Fractions

A fraction has two parts that need to be separated from each other, namely the numerator and denominator. Customarily this is done in print by placing the numerator centered over a horizontal line, under which the denominator is centered. Take

$$
\frac{x-3}{x+5-y}
$$

as an example. Print readers have the benefit of seeing complicated numerators and denominators automatically delimited by this notation. Dr. Nemeth wanted to afford braille readers the same convenience without phantom parentheses. Moreover he wanted readers to know that they were dealing with a fraction right from the get-go which is another convenience afforded by print notation. Thus he kept his code consistent with Non Enclosure Principle and the Just in Time Information Principle. Therefore, he opens a fraction with a begin fraction indicator $::$ (th sign) which we say as "b-frac". He switches to the denominator in the same way as the fraction line switches the print reader. He does it with an over sign $!!$ (st sign) which we say as "over". Finally, he concludes the fraction with an end fraction sign $\vdots:$ which we say as "e-frac". At this point I bet you're thinking, "Isn't the cell for e-frac the same as the umber sign?" Well, it is. Since braille has 64 possible cells including the space, some have to get reused occasionally. Besides, there is no danger of a reader thinking that they are actually looking at a level indicator in front of a number, as the e-frac sign has something immediately on its left. Let's see a few examples.

## Example

- Print Math: $\frac{1}{x+1}$
- MathSpeak: "b-frac one over x plus one e-frac"
- Braille: :! : : : : : : : : : : :


## Example

- Print Math: $\frac{a+b+c-e}{x+1}$
- MathSpeak: "b-frac a plus b plus c minus e over x plus one e-frac"


Now we can embed fractions within fractions, but this does something drastic to the print; it makes the outer fraction large and in charge! These outer fractions are called complex fractions and fractions that contain complex fractions are called hyper-complex fractions. Much beyond that level of embedding we need to use more sophisticate spatial Nemeth Code notation, and so this is beyond the scope of this pamphlet. Outside of Real-Analysis, however, such beasts are very rare. We want to let the braille reader know in advance how big the beast will get, and also which is the big overline. This is critical to understanding the complex fraction. So we modify the notation of the outer fraction by putting cells with dot 6 only before the b -frac, over and e-frac indicators. We say these indicators as b-b-frac o-over e-e-frac. Hyper complex fractions are amended in a similar way as the table below shows:

| Type | Beginning indicator | Over indicator | End indicator |
| :--- | :--- | :--- | :--- |
| Fraction | b-frac | Over | e-frac |
|  | $\vdots \vdots$ | $\vdots \vdots$ | $\vdots \vdots$ |
| Complex Fraction | b-b-frac | o-over | e-e-frac |
|  | $\vdots \vdots:$ | $\vdots \vdots \vdots$ | $\vdots \vdots \vdots$ |
|  | b-b-b-frac | o-o-over | e-e-e-frac |
|  | $\vdots \vdots \vdots: \vdots$ | $\vdots \vdots \vdots \vdots$ | $\vdots \vdots \vdots \vdots$ |

The fraction notation of fractions inside a complex fraction or hyper-complex fraction does not change otherwise. Some examples:

## Example

- Print Math: $\frac{1+\frac{1}{x}}{x+\frac{1}{y}+y}$
- MathSpeak: "b-b-frac one plus b-frac one over x e-frac o-over x plus b-frac one over y e-frac plus y e-e-frac"
- Braille:


## Example

- Print Math: $\frac{1}{1+\frac{1}{1+\frac{1}{2}}}$
- MathSpeak: "b-b-b-frac one o-o-over one plus b-b-frac one o-over one plus b-frac one over two e-frac e-e-frac e-e-e-frac"



## Exercises

Say and braille the following:

1. $\frac{1}{2}$
2. $\frac{2 x-3}{k+1}$
3. $1+\frac{1}{2 x}$
$4 x+2$
4. $\frac{\frac{1}{2}}{\frac{1}{3}}$

## Radicals

At last a chance to inflict a little Latin on you the unsuspecting reader! The term "radical" comes from radix, which is Latin for root. They are called this because they are used to represent roots mathematically. That is they are synonymous with fractional powers. The other bit of Latin that is germane to this discussion is vinculum, which means fetter. The vinculum is the characteristic sign of a radical, e.g.: $\sqrt{ }$ the part that goes under the vinculum is called the radicand (more Latin for "the thing that it to be rooted") and the part that sits on the lip is called the index of the radical

$$
\sqrt[i n d e x]{\text { radicand }}
$$

When we speak a radical without a index aloud we say "rad-whatever the radicand is-erad" the rad cell is the ar sign or $:!$. E-rad is the er sign or $::$. If on the other hand the radical has an index we say "index", state the index, then say rad and continue. The index indicator is $:$ : in braille.

## Example

- Print Math: $\sqrt{2}$
- MathSpeak: "rad two e-rad"
- Braille: :: : : : :


## Example

- Print Math: $\sqrt[3]{\frac{1}{x+1}}$
- MathSpeak: "index three rad b-frac one over x plus one e-frac e-rad "
- Braille: : : : : : : : : : : : : : : : : : : : : : : :

Notice that we have embedded a fraction within a radical without a hitch. There is a recipe for embedding radical within radicals. Dr. Nemeth asserts that it is indeed one of the transgressions of his youth that shall be remedied in UBS. He should have organized radical embedding like fraction embedding so that the reader knew immediately how big the radical structure was right off. Instead he appends indicators as you go deeper into the embedding. The depth indicator is $:!$, and if it is on a rad then you say rad-rad or rad-rad-rad depending on how deep your inner radical is. Similarly you say e-e-rad or e-e-rad, and i-index or i-index, again depending on depth.

## Example

- Print Math: $\sqrt{1+\sqrt{2}}$
- MathSpeak: "rad one plus rad-rad 2 e-e-rad e-rad"
- Braille: : : : : : : : : : : : : : : : :


## Example

- Print Math: $\sqrt[3]{1+\sqrt[4]{1+\sqrt[5]{2}}}$
- MathSpeak: "index 3 rad one plus i-index four rad-rad one plus i-i-index five rad-radrad 2 e-e-e-rad e-e-rad e-rad"
- Braille: ! : : : : : : : :

Radicals may be embedded like this to any desired depth.

## Exercises

Say and braille the following:

1. $\sqrt[3]{x+2}$
2. $1+\sqrt{1-\sqrt{2}}$
3. $\sqrt[x+y]{x+\sqrt[x-y]{y}}$

## Scripts

Scripts refer to superscripts and subscripts. While the majority of scripts that will be encountered in high school algebra are going to be superscripts students are introduced to subscripts as indices and possibly multiply embedded scripts. In any case there are two principal types of scripts, the superscript, e.g. $x^{2}$, and the subscript, e.g. $a_{j}$. I didn't call the superscript an "exponent" for a good reason. The superscript notation is used in contexts other than algebra, and it can mean various things depending on that context.

The point of the Nemeth Code is to give the braille reader an accurate picture of what the print looks like as the Be True to the Print Principle states. For a superscript we say "sup" (pronounced soup) and use the braille indicator :! We say sub for a subscript and indicate it with :: And if we return to the baseline, we say "base" put a symbol in that looks like :. If the print script is terminated by white space or a sign of comparison like equals sign then a baseline indicator is not needed. Otherwise You need to use it lest the reader get confused about the actual contents of the script. Note that $x^{2}+x+1$ is very different from $x^{2+x}+1$ ! Let's see some examples

## Example

- Print: $x^{2}$
- MathSpeak: "x sup two "

Braille: :!: :: : :

## Example

- Print: $x_{a}$
- MathSpeak: "x sub a"

Braille: :!: :!:

## Example

- Print: $x^{2}+x+1$
- MathSpeak: "x sup two base plus x plus one"
- Braille: : : : : : : : : : : : : : : : : :


## Example

- Print: $x^{2+x}+1$
- MathSpeak: "x sup two plus $x$ base plus one"
- Braille: : : : : : : : : : : : : : : : : : :

When a base has both a superscript and a subscript, the subscript is read first, and then the superscript is read. No return to baseline occurs in between them and thus no baseline indicator is read. Note that the symbol $\int$ is read as "integral" and has as its braille cell $\vdots$ :

## Example

- Print: $\int_{0}^{1} x^{2} d x=\frac{1}{3}$
- MathSpeak: "integral sub zero sup one base x sup two base $\mathrm{d} x$ equals $b$-frac one over three e-frac"


Now following the Just in Time Information Principle, we want to explicitly indicate the level that a script is at any given point we do this by telling the level relative to the base line, out-side-in in terms of embedding. That is, sup and sub are uttered in strings that tell the reader how to get to that level following a trail of scripts starting at the baseline. So if you see a subscript embedded in a superscript then that is read as "sup sub" if you see a superscript of a superscript then that is read "sup sup".

## Example

- Print: $x^{y_{j}+z_{j}}+1$
- MathSpeak: "x sup y sup sub $j$ sup plus z sup sub $j$ base plus one"
- Braille: : : : : : : : : : : : : : : : : : : : : : : : : : : : : : : :

As you can see above, the level is reported relative to the base line every time it changes.
There are two additional concerns with superscripts and subscripts. The first is purely numerical subscripts such as you would see in chemistry. These do not require subscript indication $\mathrm{C}_{6} \mathrm{H}_{12} \mathrm{O}_{6}$ is just read as "cap c six cap h one two cap o six". In fact, P1 requires a non-subscript indicator for the 1! The other issue is that if you see a super/sub script in a fraction which is in another super/sub script then it is treated as a second level script.

## Example

- Print: $e^{\frac{x^{2}}{z}}$
- MathSpeak: "e sup b-frac $x$ sup sup 2 over $z$ efrac "
- Braille: : : : : : : : : : : : : : : : : : : : :


## Exercises

Say and braille the following.

1. $e^{x_{j}^{2}+y_{j}^{2}}+3$
2. $x_{a}^{x_{b}}$
3. $\mathrm{H}_{2} \mathrm{O}$
4. $x^{\frac{x^{x}}{y}}$

## Stacked Structures

The last part of structural mathematics is stacked structures such as $\sum_{n=1}^{q} n^{2}$. This is read as "Stacked structure Greek cap sigma under n equals one over q terminate $n$ sup two" The stacked structure indicator is $\because$, the under indicator is $\because!$, the over indicator is $\vdots!$,
and the terminate cell is $::$. Note that $\sum_{n=1}^{q} n^{2}$ is distinct from $\sum_{n=1}^{q} n^{2}$ which uses script notation. (for the example note that infinity is $: \vdots: \%$ in braille)

## Example

- Print: $\sum_{n=1}^{\infty} \frac{1}{n^{2}}=\frac{\pi^{2}}{6}$
- MathSpeak: "Stacked structure Greek cap sigma under n equals one over infinity terminate b-frac one over n sup two base e-frac "



## Exercises

Say and braille the following:

1. $f(z)=\sum_{j=0}^{\infty} z^{j}$
2. $\quad F(\omega)=\int_{-\infty}^{\infty} f(t) e^{-2 \pi i \omega t} d t$

## References

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