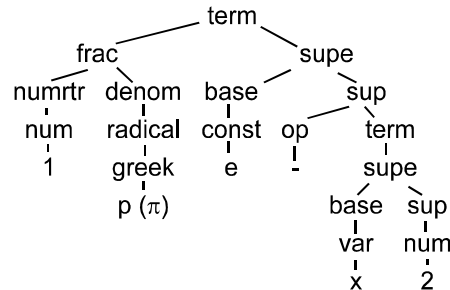


explicit notational events being the symbols themselves or other significant glyphs such as fraction lines. For example, see how the following print notation makes the corresponding tree structure obvious.

$$\frac{1}{\sqrt{\pi}} e^{-x^2}$$



Note how the implicit notational events guide your eye to the correct tree structure. The process that you go through as you analyze the tree structure is called parsing, and being able to do it correctly is absolutely essential to your understanding of the mathematical content represented by that print expression. As one can imagine using print like shifts in braille won't work. Fingers do not have the eye's capacity to scan two dimensionally. Thus dots that get shrunk and raised just look like a damaged braille page.

In his contribution to Braille for the Millennium Dr. Nemeth (2001) outlined his design principles for the Nemeth Code. As some of them are more faithfully implemented in the Unified Braille System, an alternative to the proposed Unified English Braille Code, I have selected the few for presentation that I think best characterize the 1972 revision. They are quoted editorially as follows:

- 1) **Non Enclosure Principle:** Don't put any phantom parentheses or other enclosures into the braille that are not in the print.
- 2) **Just In Time Information Principle:** Give the reader the needed information exactly when it is needed--Don't make him/her have to go looking for it
- 3) **Be True To The print Principle:** Don't make any braille notation that does not correspond to the print notation. I.E. don't make the braille notation dependant on the meaning of an expression; let the reader infer that from his/her knowledge base just as a print reader would.
- 4) **Good Mnemonics Principle:** Give the reader symbols that are grouped together logically when the print symbols are grouped together logically. Preserve Symmetry of notation.
- 5) **Continuous Notation Principle:** Don't interrupt the reader's reading with letter signs and number signs unless some specific indication is needed.

We will explore these ideas in detail, as they become relevant to us.

Braille Basics

At the level of cells Nemeth code looks like any other six-cell braille. As I am hoping that this pamphlet will be shared with traditional teachers of mathematics, I felt it necessary to

However, as students progress through Algebra we more commonly see a \cdot as the multiplication indicator. We call this “dot-times”

Example

- Print Math: 5×3
- MathSpeak: "five dot-times three"
- Braille: \cdot (or \cdot if the level is understood)

In Pre-Algebra a \div is used to indicate simple division tasks. We call this “inline-divide”.

Example

- Print Math: $5 \div 3$
- MathSpeak: "five inline-divide three"
- Braille: \div (or \div if the level is understood)

Although I discourage my print students from using slash notation for division because it is inherently ambiguous, it is simply read as “slash” if encountered in a text.

Example

- Print Math: $5/3$
- MathSpeak: "five slash three"
- Braille: $/$ (or $/$ if the level is understood)

If you encounter any signs that I have not treated in this text I refer you to the code specification (Nemeth, 1972). <http://www.tsbvi.edu/math/nemeth-reference.htm> is an excellent cheat sheet prepared by Susan Osterhaus at the Texas School for the Blind and Visually Impaired.

Exercises

Say and braille the following:

1. $1+x+y$
2. $2.5-6.7$
3. $-298,475$
4. $\sin\theta + \sin\alpha$
5. $28.6 \times 5 \div x$

Signs of Comparison

One of the basic structures in print mathematics is the equals sign. In fact, you can't technically have an equation without one. When we see an equals sign similar notation such as a greater than sign, or a less than or equal to sign, we simply repeat that sign as if it were in English. If you want to you can abbreviate less than or equal to by saying

"less-eeek" be sure to explain what you are doing to your student, however. In braille these signs are four or five cell constructions. The first cell in the construction is always a space, and the last cell is always a space. This is to distinguish them from Greek letters and the like. For instance, the equals sign without the surrounding spaces would be the Greek letter kappa! Therefore, numbers that come after a sign of comparison need the level indicating number sign. Here, I was planing to put a table of the signs, but to see how they work with the spaces, it is best to just go through some examples.

Example

- Print Math: $x = 1$
- MathSpeak: "x equals one"
- Braille: ⠠ ⠨ ⠨ ⠨ ⠨

Note the number sign on the braille 1.

Example

- Print Math: $x < y$
- MathSpeak: "x is less than y"
- Braille: ⠠ ⠨ ⠨ ⠨

Note the similarity between the print less-than sign and the braille one. Greater than is similar. Now if we want to express the or equal to part, we stick the cell ⠨ before the last space.

Example

- Print Math: $2x + 5 \geq 6$
- MathSpeak: "two x plus five is greater than or equal to 6"
- Braille: ⠠ ⠨ ⠨ ⠨ ⠨ ⠨ ⠨ ⠨ ⠨

Be careful with the greater than sign because out of context it looks like a ".1".

I will leave you the exercises to discover what less than or equal to, and greater than by itself look like. I defer a discussion of more complicated signs of comparison to the Nemeth Code book as they appear very rarely in secondary math.

Exercises

Say and braille the following:

1. $2x = 3$
2. $y - 7 > 6$
3. $-1 \leq \cos x \leq 1$
4. $5 > x \geq 2$
5. $\kappa = \kappa$ (I couldn't resist!)

Enclosures

The last things that we need to be able to do any in-line math are signs of enclosure. There are three that are commonly used: () parentheses, [] brackets, and {} braces. As there is a left and a right form of each of these, when we are speaking them we say "l-paren" for left parenthesis and "r-paren" for a right parenthesis. Similarly we say things

like "l-bracket" or "r-brace". Don't refer to them as "open parens" because mathematicians have this bad habit of opening an enclosure with a right parenthesis and closing it with a left bracket as a means of pure notational abuse. The l-paren is the cell ⠠ and the r-paren is the cell ⠡. We make brackets and braces by sticking a bracket or brace indicator before a respective l-paren or r-paren. As part of the Just in Time Information Principle these indicators always come before, even though it would be more visually balanced for them to come after. So the cell patterns are:

[]	{	}
⠠	⠡	⠠	⠡

Let's see some examples of these enclosures in action!

Example

- Print Math: $(2x + 5)$
- MathSpeak: "l-paren two x plus five r-paren"
- Braille: ⠠⠠⠠⠠⠠⠠

Example

- Print Math: $3[6 - (2x + 5)]$
- MathSpeak: "three l-bracket six minus l-paren two x plus 5 r-paren r-bracket"
- Braille: ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

Example

- Print Math: $\{1, a, cow\}$
- MathSpeak: "l-brace one comma a comma c o w r-brace"
- Braille: ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

Note the spaces added so as not to confuse the reader regarding commas in a number versus commas separating list items. Number signs need to be used in the case that numbers with spaces before them are in the group.

Example

- Print Math: $]0,1]$
- MathSpeak: "l-bracket zero comma 1 l-bracket"
- Braille: ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

Again, if you encounter more complicated signs of enclosure, I refer you to the Nemeth Code Book or to the TSBVI site.

Exercises

Say and braille the following:

1. $(2x - 1)$
2. $1 - [1 - (1 - x)]$
3. $\{a, aa, aaa, b, ab, aab, aaab\}$

Structural Math Notation

Fractions

A fraction has two parts that need to be separated from each other, namely the numerator and denominator. Customarily this is done in print by placing the numerator centered over a horizontal line, under which the denominator is centered. Take

$$\frac{x - 3}{x + 5 - y}$$

as an example. Print readers have the benefit of seeing complicated numerators and denominators automatically delimited by this notation. Dr. Nemeth wanted to afford braille readers the same convenience without phantom parentheses. Moreover he wanted readers to know that they were dealing with a fraction right from the get-go which is another convenience afforded by print notation. Thus he kept his code consistent with Non Enclosure Principle and the Just in Time Information Principle. Therefore, he opens a fraction with a begin fraction indicator ⠠ (th sign) which we say as "b-frac". He switches to the denominator in the same way as the fraction line switches the print reader. He does it with an over sign ⠨ (st sign) which we say as "over". Finally, he concludes the fraction with an end fraction sign ⠡ which we say as "e-frac". At this point I bet you're thinking, "Isn't the cell for e-frac the same as the number sign?" Well, it is. Since braille has 64 possible cells including the space, some have to get reused occasionally. Besides, there is no danger of a reader thinking that they are actually looking at a level indicator in front of a number, as the e-frac sign has something immediately on its left. Let's see a few examples.

Example

- Print Math: $\frac{1}{x + 1}$
- MathSpeak: "b-frac one over x plus one e-frac"
- Braille: ⠠⠠⠨⠠⠠⠡

